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Remarks on the Qin-Ma Parametrization of Quark Mixing Matrix

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Abstract

Recently, Qin and Ma (QM) have advocated a new Wolfenstein-like parametrization of the quark mixing matrix based on the triminimal expansion of the Cabibbo-Kobayashi-Maskawa (CKM) parametrization. The *CP*-odd phase in the QM parametrization is around 90° just as that in the CKM parametrization. We point out that the QM parametrization can be readily obtained from the Wolfenstein parametrization after appropriate phase redefinition for quark fields and that the phase δ in both QM and CKM parametrizations is related to the unitarity angles α, β and γ, namely, $\delta = \beta + \gamma$ or $\pi - \alpha$. We show that both QM and Wolfenstein parametrizations can be deduced from the CKM and Chau-Keung-Maiani ones. By deriving the QM parametrization from the exact Fritzsch-Xing (FX) parametrization of the quark mixing matrix, we find that the phase of the FX form is in the vicinity of $-270°$ and hence $\sin\delta \approx 1$. We discuss the seeming discrepancy between the Wolfenstein and QM parametrizations at the high order of $\lambda \approx |V_{us}|$.

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1. In the standard model with three generations of quarks, the 3×3 unitary quark mixing matrix

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{1}$$

can be parametrized in infinitely many ways with three rotation angles and one CP-odd phase. All different parametrizations lead to the same physics. A well-known simple parametrization introduced by Wolfenstein [1] is

$$V_{\rm Wolf} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{2}$$

Using the global fits to the data, the four unknown real parameters A, λ, ρ and η are determined to be

$$A = 0.812^{+0.013}_{-0.027}\,, \quad \lambda = 0.22543 \pm 0.00077\,, \quad \bar\rho = 0.144 \pm 0.025\,, \quad \bar\eta = 0.342^{+0.016}_{-0.015}\,, \tag{3}$$

by the CKMfitter Collaboration [2] and

$$A = 0.807 \pm 0.01\,, \quad \lambda = 0.22545 \pm 0.00065\,, \quad \bar\rho = 0.143 \pm 0.03\,, \quad \bar\eta = 0.342 \pm 0.015\,, \tag{4}$$

by the UTfit Collaboration [3], where $\bar\rho = \rho(1 - \lambda^2/2 + \cdots)$ and $\bar\eta = \eta(1 - \lambda^2/2 + \cdots)$.

Recently, Qin and Ma (QM) [4] have advocated a new Wolfenstein-like parametrization of the quark mixing matrix

$$V_{\rm QM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & h\lambda^3 e^{-i\delta_{\rm QM}} \\ -\lambda & 1-\lambda^2/2 & (f + he^{-i\delta_{\rm QM}})\lambda^2 \\ f\lambda^3 & -(f + he^{i\delta_{\rm QM}})\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{5}$$

based on the triminimal expansion of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. It is obvious that once the parameters f and h are fixed from the matrix elements V_{td} and V_{ub}, respectively, the phase $\delta_{\rm QM}$ is ready to be determined from the measurement of V_{cb}. From the global fits to the quark mixing matrix given below in Eq. (8) we obtain

$$f = 0.749^{+0.034}_{-0.037}\,, \qquad h = 0.309^{+0.017}_{-0.012}\,, \qquad \delta_{\rm QM} = (89.6^{+2.94}_{-0.86})^\circ\,. \tag{6}$$

Therefore, CP violation is approximately maximal in the sense that $\sin\delta \approx 1$. Indeed, it is known that the phase in the Kobayashi-Maskawa parametrization is also in the vicinity of maximal CP violation. We shall show below that by rephasing the Wolfenstein parametrization, it is easily seen why the phase $\delta_{\rm QM}$ of the Qin-Ma parametrization and $\delta_{\rm KM}$ of the Kobayashi-Maskawa one are both of order 90°.

Since the phases of the matrix elements V_{ub} and V_{td} in the Wolfenstein parametrization are $\arctan(\eta/\rho) \approx \gamma$ and $\arctan(\eta/(1-\rho)) \approx \beta$, respectively, it has been argued in [4] that "one has difficulty to arrive at the Wolfenstein parametrization from the triminimal parametrization of the KM matrix". The purpose of this short note is to point out that both Wolfenstein and Qin-Ma parametrizations can be obtained easily from the Cabibbo-Kobayashi-Maskawa and Chau-Keung-Maiani matrices to be discussed below. Koide [5] pointed out that among the possible

parametrizations of the quark mixing matrix, only the CKM and the Fritzsch-Xing (FX) [6] parametrizations can allow to have maximal *CP* violation. In this work, we are going to show that the QM parametrization derived from the FX parametrization will enable us to see the feature of maximal *CP* nonconservation in the FX form. We shall also compare the Wolfenstein and QM parametrizations at the high order of λ.

2. The well-known Cabibbo-Kobayashi-Maskawa parametrization of V is given by [7]

$$V_{\rm CKM} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{\rm KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{\rm KM}} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{\rm KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{\rm KM}} \end{pmatrix}, \tag{7}$$

where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$. Using the matrix elements of $|V|$ determined from global fits at 1σ level [2]

$$\begin{pmatrix} 0.97425 \pm 0.00018 & 0.22543^{+0.00077}_{-0.00077} & 0.00354^{+0.00016}_{-0.00014} \\ 0.22529 \pm 0.00077 & 0.97342^{+0.00021}_{-0.00019} & 0.04128^{+0.00058}_{-0.00129} \\ 0.00858^{+0.00030}_{-0.00034} & 0.04054^{+0.00057}_{-0.00129} & 0.999141^{+0.000053}_{-0.000024} \end{pmatrix}, \tag{8}$$

we obtain

$$\theta_1 = (13.03 \pm 0.05)^\circ, \quad \theta_2 = (2.18^{+0.08}_{-0.09})^\circ \quad \theta_3 = (0.90^{+0.044}_{-0.039})^\circ, \quad \delta_{\rm KM} = (88.88^{+4.11}_{-2.05})^\circ. \tag{9}$$

There is one disadvantage in this parametrization, namely, the matrix element V_{tb} has a large imaginary part. Since *CP*-violating effects are known to be small, it is thus desirable to parameterize the mixing matrix in such a way that the imaginary part appears with a smaller coefficient. The parametrization proposed by Maiani in 1977 [8]

$$V_{\rm Maiani} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\phi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\phi} & s_{23}c_{13}e^{i\phi} \\ s_{12}s_{23}e^{-i\phi} - c_{12}c_{23}s_{13} & -c_{12}s_{23}e^{-i\phi} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix} \tag{10}$$

has the nice feature that its imaginary part is proportional to $s_{23}\sin\phi$, which is of order 10^{-2}. In 1984 Chau and Keung introduced another parametrization [9] (see also [10])

$$V_{\rm CK} = V_{\overline{\rm CKM}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\phi} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\phi} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\phi} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\phi} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\phi} & c_{23}c_{13} \end{pmatrix}, \tag{11}$$

which is equivalent to the Maiani parametrization after the quark field redefinition: $t \to t\,e^{i\phi}$ and $b \to b\,e^{-i\phi}$. It is evident that the imaginary part in this parametrization is proportional to $s_{13}\sin\phi$, of order 10^{-3}. This Chau-Keung-Maiani (another CKM !) parametrization denoted by $V_{\overline{\rm CKM}}$ or $V_{\rm CK}$ has been advocated by the Particle Data Group (PDG) [11] to be the standard parametrization for the quark mixing matrix. [1] It follows from Eqs. (8) and (11) that

$$\theta_{12} = (13.03 \pm 0.05)^\circ, \quad \theta_{23} = (2.37^{+0.03}_{-0.07})^\circ, \quad \theta_{13} = (0.20^{+0.01}_{-0.01})^\circ, \quad \phi = (67.19^{+2.40}_{-1.76})^\circ\ . \tag{12}$$

[1] The Maiani parametrization Eq. (10) was once proposed by PDG (1986 edition) [12] to be the standard parametrization for the quark mixing matrix.

The Wolfenstein parametrization can be easily obtained from the exact $\overline{\text{CKM}}$ parametrization by using the relations

$$s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{-i\phi} = A\lambda^3(\rho - i\eta). \tag{13}$$

It should be stressed that the Wolfenstein parametrization given in Eq. (2) is just an approximation to order λ^3 and sometimes it may give a wrong result if higher order λ terms are not included. For example, to $\mathcal{O}(\lambda^3)$ the rephasing-invariant quantity $F = V_{ud}V_{cs}V_{us}^*V_{cd}^*$ is real and hence cannot induce CP violation. In order to obtain the imaginary part of F, one has to expand Eq. (2) to the accuracy of $\mathcal{O}(\lambda^5)$ (see Eq. (33) below). To derive the Wolfenstein parametrization from the CKM one, we first rotate the phases of some of the quark fields $s \to s\,e^{i\pi}$, $c \to c\,e^{i\pi}$, $b \to b\,e^{-i(\theta+\pi)}$, $t \to t\,e^{-i(\delta_{\rm KM}-\theta)}$ and substitute the relations

$$s_1 = \lambda, \qquad s_2 e^{-i(\delta_{\rm KM}-\theta)} = A\lambda^2(1-\rho-i\eta), \qquad s_3e^{-i\theta} = A\lambda^2(\rho - i\eta) \tag{14}$$

in the CKM parametrization to obtain the Wolfenstein one. From the above equation we are led to

$$\delta_{\rm KM} = \arctan\left(\frac{\eta}{\rho}\right) + \arctan\left(\frac{\eta}{1-\rho}\right) \approx \gamma + \beta = \pi - \alpha, \tag{15}$$

where the three angles α, β and γ of the unitarity triangle are defined by

$$\alpha \equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \quad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \quad \gamma \equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right), \tag{16}$$

and they satisfy the relation $\alpha + \beta + \gamma = \pi$. Since $\alpha = (91.0 \pm 3.9)^\circ$, $\beta = (21.76^{+0.92}_{-0.82})^\circ$ and $\gamma = (67.2 \pm 3.0)^\circ$ [2], the phase $\delta_{\rm KM}$ is thus very close to 90°.

It is also easily seen that the phase $\delta_{\rm QM}$ of the Qin-Ma parametrization can be expressed in terms of the unitarity angles α, β and γ. Starting from the Wolfenstein parametrization in Eq. (2) with $V_{ub} = |V_{ub}|e^{-i\gamma}$ and $V_{td} = |V_{td}|e^{-i\beta}$, one can rephase the t and b quark fields as $t \to t\,e^{i\beta}$, $b \to b\,e^{-i\beta}$. Substituting the relations

$$A\sqrt{(1-\rho)^2 + \eta^2} = f, \qquad A\sqrt{\rho^2+\eta^2} = h, \qquad Ae^{-i\beta} = f + he^{-i\delta_{\rm QM}}, \tag{17}$$

in the Wolfenstein parametrization, we see that the QM parametrization is obtained with the phases of V_{ub} and V_{cb} expressed as

$$\delta_{\rm QM} = \gamma + \beta = \pi - \alpha\ , \qquad \arctan\Big(\frac{h\sin\delta_{\rm QM}}{f + h\cos\delta_{\rm QM}}\Big) = \beta\ . \tag{18}$$

Therefore, the phases $\delta_{\rm QM}$ and $\delta_{\rm KM}$ are both equal to $\gamma + \beta$ or $\pi - \alpha$. Note that Eq. (17) leads to $A = (f^2+h^2)^{1/2}$ and $A(1-2\rho+2\rho^2+2\eta^2)^{1/2} = (f^2+h^2)^{1/2}$. These two relations are consistent with each other as $(1-2\rho+2\rho^2+2\eta^2)^{\frac{1}{2}} = 0.9993$ is very close to 1.

It is straightforward to obtain the Qin-Ma parametrization from the CKM matrix by making the phase rotation $s \to s\,e^{i\pi}$, $c \to c\,e^{i\pi}$, $b \to b\,e^{i(\pi-\delta_{\rm KM})}$, followed by the replacement

$$s_1 = \lambda, \qquad s_2 = f\lambda^2, \qquad s_3e^{-i\delta_{\rm KM}} = h\lambda^2 e^{-i\delta_{\rm QM}}. \tag{19}$$

As a result, $\delta_{\rm QM} = \delta_{\rm KM}$, as it should be. For the $\overline{\text{CKM}}$ matrix, the Qin-Ma parametrization is obtained by first performing the quark field redefinition $b \to b\,e^{-i\theta}$ and $t \to t\,e^{i\theta}$ and then adapting the relations

$$s_{12} = \lambda, \qquad s_{23}e^{-i\theta} = (f + he^{-i\delta_{\rm QM}})\lambda^2, \qquad s_{13}e^{-i(\phi+\theta)} = h\lambda^3 e^{-i\delta_{\rm QM}}. \tag{20}$$

From Eqs. (13), (14), (19) and (20) we see that

$$
\begin{aligned}
\frac{s_{13}}{s_{23}} &= \sqrt{\rho^2+\eta^2}\,\lambda = \frac{h}{\sqrt{f^2+h^2}}\lambda = 0.38\lambda\,, \\
\frac{s_3}{s_2} &= \frac{h}{f} = \frac{\sqrt{\rho^2+\eta^2}}{\sqrt{(1-\rho)^2+\eta^2}} = 0.41\,,
\end{aligned} \tag{21}
$$

and

$$
s_1 : s_2 : s_3 = 1 : 0.75\lambda : 0.31\lambda\,, \qquad s_{12} : s_{23} : s_{13} = 1 : 0.81\lambda : 0.31\lambda^2. \tag{22}
$$

Therefore, the hierarchial pattern for the three mixing angles in the Cabibbo-Kobayashi-Maskawa and Chau-Keung-Maiani parametrizations is very similar for the first two angles but different for the third angle. The corresponding Jarlskog invariant J [13] has the expression [2]

$$
\begin{aligned}
J_{\rm CKM} &\approx s_1^2 s_2 s_3 \sin\delta_{\rm KM} = fh\lambda^6, \\
J_{\overline{\rm CKM}} &\approx s_{12}s_{23}s_{13}\sin\phi = A^2\eta\lambda^6,
\end{aligned} \tag{23}
$$

with the magnitude of 3.0×10^{-5}.

3. Fritzsch and Xing [14] were the first (see also [15]) to point out that there exist nine fundamentally different ways to describe the quark mixing matrix. [3] Moreover, they argued that the Fritzsch-Xing (FX) parametrization proposed by them [6]

$$
V_{\rm FX} = \begin{pmatrix} s_x s_y c_z + c_x c_y e^{-i\phi_{\rm FX}} & c_x s_y c_z - s_x c_y e^{-i\phi_{\rm FX}} & s_y s_z \\ s_x c_y c_z - c_x s_y e^{-i\phi_{\rm FX}} & c_x c_y c_z + s_x s_y e^{-i\phi_{\rm FX}} & c_y s_z \\ -s_x s_z & -c_x s_z & c_z \end{pmatrix}, \tag{24}
$$

in which the *CP*-violating phase resides solely in the light quark sector, stands up as the most favorable description of the flavor mixing. As shown in [5], among the nine distinct parametrizations, only the CKM and FX parametrizations allow to have maximal *CP* violation. To see this is indeed the case for the FX form, let us derive the QM parametrization from it.

Substituting the relations

$$
s_x = \frac{f}{\sqrt{f^2+h^2}}\,\lambda, \quad s_y = \frac{h}{\sqrt{f^2+h^2}}\,\lambda, \quad s_z = \sqrt{f^2+h^2}\,\lambda^2, \tag{25}
$$

in the FX parametrization leads to

$$
V_{\rm FX} = \begin{pmatrix} (1-\lambda^2/2)e^{-i\phi_{\rm FX}} & \lambda e^{i\theta_{\rm FX}} & h\lambda^3 \\ -\lambda e^{-i(\phi_{\rm FX}+\theta_{\rm FX})} & 1-\lambda^2/2 & \sqrt{f^2+h^2}\,\lambda^2 \\ -f\lambda^3 & -\sqrt{f^2+h^2}\,\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4), \tag{26}
$$

with

$$
\sin\theta_{_{\rm FX}} = \frac{f}{\sqrt{f^2+h^2}}\sin\phi_{_{\rm FX}}, \qquad \cos\theta_{_{\rm FX}} = \frac{h}{\sqrt{f^2+h^2}}\,(1-\frac{f}{h}\cos\phi_{_{\rm FX}}). \tag{27}
$$

[2] The concept of rephasing invariance for physical quantities and the use of the rephasing invariant quantity J became popular in the early and middle eighties. Historically, Chau and Keung already pointed out in their 1984 seminal paper that all *CP*-violating effects are proportional to a universal factor which they called X_{CP} [9]. They showed explicitly that the quantity ${\rm Im}[V_{ij}V_{kl}V_{il}^*V_{kj}^*]$ is proportional to X_{CP}.

[3] Of course, the freedom of rotating the phase of quark fields will render the parametrization of the quark mixing matrix infinitely many.

Then, making the quark field redefinition

$$u \to u^{i\phi_{\rm FX}}, \quad c \to c\, e^{i(\phi_{\rm FX}+\theta_{\rm FX})}, \quad t \to t\, e^{-i\pi}, \quad s \to s\, e^{-i(\phi_{\rm FX}+\theta_{\rm FX})}, \quad b \to b\, e^{i\pi}, \tag{28}$$

and setting

$$\phi_{\rm FX} = -(\delta_{\rm QM} + \pi), \tag{29}$$

we finally arrive at the Qin-Ma parametrization (5). Since $\delta_{\rm QM}$ is around 90°, it is clear that the phase $\phi_{\rm FX}$ in the vicinity of -270° leads to maximal *CP* violation with $\sin\phi_{\rm FX} = 1$. The corresponding Jarlskog invariant is

$$J_{\rm FX} = s_x s_y s_z^2 \sin\phi_{\rm FX} = fh\lambda^6\ . \tag{30}$$

From Eq. (25) we obtain

$$s_x : s_y : s_z = 1 : \frac{h}{f} : \frac{f^2+h^2}{f}\lambda = 1 : 0.41 : 0.88\lambda\,. \tag{31}$$

As a consequence, the hierarchical pattern for the mixing angles in the FX parametrization differs from that in CKM and $\overline{\rm CKM}$ ones. Recall that the parameter λ is equal to s_1 (s_{12}) in the CKM ($\overline{\rm CKM}$) parametrization, while it is identical to $\sqrt{s_x^2+s_y^2}$ in the FX parametrization. From Eq. (8) we obtain the mixing angles

$$\theta_x = (11.95^{+0.83}_{-0.64})^\circ, \qquad \theta_y = (4.90^{+0.39}_{-0.26})^\circ, \qquad \theta_z = (2.38^{+0.07}_{-0.03})^\circ. \tag{32}$$

4. In future experiments such as LHCb and Super B ones, more precise measurements of the CKM matrix elements are expected so that high order λ terms of the CKM matrix elements become more important. In principle, the expression of the Wolfenstein and QM parametrizations to the high order of λ can be obtained from the exact parametrization of the quark mixing matrix by expanding it to the desired order of λ. For example, the substitution of the relations (13) in the $\overline{\rm CKM}$ matrix for the Wolfenstein parametrization [16] and relations (19) in the CKM matrix for the QM parametrization [4] lead to

$$\begin{aligned}
V_{\rm Wolf} &= \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5(1-2\rho-2i\eta) & 1-\frac{\lambda^2}{2}-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta)+\frac{1}{2}A\lambda^5(\rho+i\eta) & -A\lambda^2+\frac{1}{2}A\lambda^4(1-2\rho-2i\eta) & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix} \\
&\quad +\mathcal{O}(\lambda^6)\ , \\
V_{\rm QM} &= \begin{pmatrix} 1-\frac{\lambda^2}{2}-\frac{\lambda^4}{8} & \lambda-\frac{h^2\lambda^5}{2} & h\lambda^3 e^{-i\delta_{\rm QM}} \\ -\lambda+\frac{f^2\lambda^5}{2} & 1-\frac{\lambda^2}{2}-\frac{1}{8}(1+4f^2+8fhe^{i\delta_{\rm QM}}+4h^2)\lambda^4 & (f+he^{-i\delta_{\rm QM}})\lambda^2-\frac{1}{2}h\lambda^4 e^{-i\delta_{\rm QM}} \\ f\lambda^3 & -(f+he^{i\delta_{\rm QM}})\lambda^2+\frac{1}{2}f\lambda^4 & 1-\frac{1}{2}(f^2+2fhe^{-i\delta_{\rm QM}}+h^2)\lambda^4 \end{pmatrix} \\
&\quad +\mathcal{O}(\lambda^6)\ ,
\end{aligned} \tag{33}$$

up to the order of λ^5. However, care must be taken when one compares higher order terms in two different parametrizations. The point is that when λ is treated as an expansion parameter for the quark mixing matrix, the other parameters should be of order unity. We know this is not the case in reality: the parameters h, ρ and η are of order λ numerically, while A, f and δ are of order unity. This fact leads to the seeming discrepancy between the corresponding elements of $V_{\rm Wolf}$ and

$V_{\rm QM}$. For instance, taking into account h, ρ and η being of order λ numerically, the order λ^5 terms in V_{us} of $V_{\rm QM}$ and in V_{td} of $V_{\rm Wolf}$ are effectively of order λ^7 and λ^6 being negligible, respectively. Likewise, for V_{cb} of $V_{\rm QM}$, the physical (rephasing-invariant) observable $|V_{cb}|$ is obtained as

$$|V_{cb}| \approx \lambda^2 \sqrt{f^2+h^2}\ \left[1 - \frac{1}{2}\,\frac{h^2}{f^2+h^2}\ \lambda^2 + \mathcal{O}(\lambda^4)\right]\ , \tag{34}$$

where $\delta_{\rm QM} \approx 90^\circ$ has been used. With the relation $A \approx \sqrt{f^2+h^2}$, the correction to the leading term of order λ^2 starts effectively at order λ^6 being negligible. Thus, all the seeming discrepancies between the corresponding elements of $V_{\rm Wolf}$ and $V_{\rm QM}$ are resolved.

5. In this work we have shown that the Qin-Ma parametrization can be easily obtained from the Wolfenstein parametrization after appropriate phase redefinition for quark fields and that the phase δ in both QM and CKM parametrizations is related to the unitarity angles α, β and γ, namely, $\delta = \beta + \gamma$ or $\pi - \alpha$. Both QM and Wolfenstein parametrizations can be deduced from the CKM and Chau-Keung-Maiani ones. By deriving the QM parametrization from the exact Fritzsch-Xing parametrization, we find that the phase of the FX form is approximately maximal. From the analysis of this work, it is easy to see the hierarchial patterns for the quark mixing angles in various different parametrizations. Finally, we compare the Wolfenstein and QM parametrizations at the high order of λ and point out that all the seeming discrepancies between them are gone when the small parameters h, ρ and η are counted as of order λ.

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